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AN APPLICATION OF INVERSE PROBLEM TECHNIQUES TO SPATIAL STRUCTURES

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Abstract

The present paper attempts to give a definition of inverse problems for spatial structures, and to develop a design method to find the optimum configurations and shapes for such structures by applying inverse problem techniques to them. As an example, the present paper proposes a method to find the optimum shape of a shell or a space frame that has a maximum buckling capacity.

Introduction

The conventional procedure of structural analysis has normally been to find the displacements and the internal forces of a structure with given informations such as topological characters (configuration), dimensions and shape or coordinates, and material character under given loading conditions. These informations are predetermined on the basis of experiences, guess work or conjecture. Occasionally, some of them are altered when they are found unsuitable for design as a result of analysis. By this method, however, the capabilities of the structure and its materials cannot be fully exploited, especially when we design large span spatial structures, for which we have only limited experiences. To circumvent the above drawbacks of conventional design methods, it may be suitable to apply the inverse problem techniques to the design of spatial structures .

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With the development of computer science, the complex inverse problems can be solved quickly and more easily, and it seemes that the time will soon come when we can design the configuration and shape of a structure, especially a large-span spatial structure, by means of inverse porblem techniques.

1. Inverse Problems For Spatial Structure

1.1 Definition of Inverse Porblem

Many people have been trying to give the inverse problem a definition(ref. Sabatier,P.C., 1986), but there has not seem to be a unified definition for discrimination between direct and inverse problems. Before giving the definition of the inverse problems on spatial structures, we should analyze the general inverse problems mathematically. Generally, a problem can be expressed as a transformation from one space to another.

$$S \rightarrow R\,. \tag{1}$$

In the source space S, there is always a subspace Ω which is constituted by some invariable-sets with constant elements, conditions or some specific units. We call Ω the invariable space of the direct problem. So, the problem can be expressed as

$$\mathcal{F}(\Omega, \Upsilon) = R; \tag{2}$$

where $\Omega \cup \Upsilon = S$, and $\mathcal{F}$ is the tansformation, and we call such a problem a direct problem.

There are many problems in which the source subspace Ω is unknown and we have to find it by the informations of the Υ and its response R. Such a problem is called an inverse problem.

Definition : A problem is called an inverse problem if it is a problem to find its invariable space Ω of its direct problem.

1.2 The Inverse Problem For Spatial Structure

There are only few papers which give a definition of the inverse problem on spatial structures. Before defining it, the direct problem should be analyzed first. The spatial structure problems are always constituted by considering some main elements, which are

τ : Topological charactor of the structure ;

ζ : Shape, coordinates or major dimensions of the structure ;

ϵ : The material charactor of the structure ;

ρ : The prestresses (self-balancing internal forces) of the structure ;

γ : The working surroundings, temperature change

κ : The steady or dynamic (moving) loading system exerting on the structure ;

δ : The displacements, deformation of the structure ;

σ : The stresses, internal forces of the structure.

With the conventional design method, the invariable space of the problem is

$$\Omega = (\tau, \zeta, \epsilon, \rho). \tag{3}$$

The direct problem of the structural design is to find one or more elements of $\{\gamma, \kappa, \delta, \sigma\}$ corresponding to a given space Ω . For example, to find the deformation of a given structure subjected to a certern loading system is one of the direct problem, which is a transformation $\mathcal{T}(\Omega, \kappa) = \delta$. To find the buckling-load of a space frame or a shell is a direct problem $\mathcal{T}(\Omega) = \kappa$, etc. .

Contrarily, the inverse problem is a problem to find the elements of the invariable space Ω of the direct problem.

Definition : The inverse problem for spatial structure is a problem to find the elements of invariable space $\Omega = (\tau, \zeta, \epsilon, \rho)$ of its direct problem .

So, the inverse problems for spatial structures can be classified as :

1. **Topological problems** : To find the topological configuration of a structure;
2. **Shape-problems** : To find the shape, or some of the dimensions or nodal coordinates of the structure;
3. **Material-problems** : To find the best mixture of materials of some parts, some members of the structure or whole of the strucuture;
4. **Prestress-problems** : To find the optimum distribution of prestresses;

For the topological problems, the important thing is not only the mathematical analysis but also the structural design idea. These problems are challenged by many researchers, and some encouraging progress have been obtained (ref. Rozvany, G.I.N.,1989) (ref. Lin,J.H., Che,W.Y. and Yu,Y.S.,1982). Now, there are many important topics on this research, for example, optimum configuration of a space frame and multicriteria optimization with topological problem. As an example, the present paper proposes a design method for finding a space frame or a shell suitably strong against buckling.

Shape problems are these of shape-finding, shape optimization(ref.Chen,P.S., Abe,M. and Kawaguchi,M.,1993) and shape analysis of movable structures or folding-structures (ref. Kawaguchi, K., Hangai,Y. and Nabana,K.,1993). The prestress problems are always concerned with the shape-problems, because the distribution of the internal forces due to prestressing depends on the shape, and must be in equilibrium in the given shape before loading.

Now the computers have become popular and powerful enough to deal with inverse problems and we may be able to believe that there will be a revolution on the design of spatial structures in this direction. So, it is very important to develop the desgin method by application of the inverse problems.

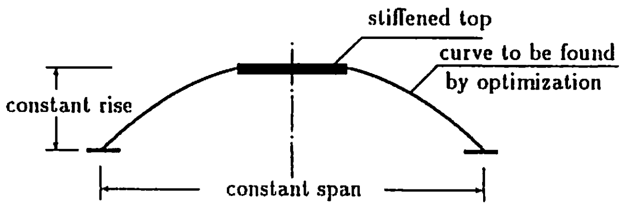

Fig.1 The section of a space frame of revolution stiffened at its top

2. Shape With Maximum Buckling-Load

2.1 Introduction of The Design Method

In design of a shell or a space frame one of the most important problems is the buckling problem, especially for large span structures. It is not rational or economical to assume a uniform stiffeness for the whole area or all of the members of the structure. We can stiffen some particular parts which are susceptible to buckling, or can be stiffened with ease. The other parts can be found by shape optimization for maximum buckling-load.

For example, a dome of revolution with a given rise can be stiffened at its top so as not to buckle (Fig.1), and the curvature of other areas can be found by optimization for maximum buckling load.

2.2 The Basic Functions

For the nonlinear analysis, by the finite difference method, finite element method or Galerkin's method, the relationship between the loads and the displacements is expressed in form, (ref. Thompson, J.M.T. and Hunt,G.W.,1973) (ref. Hangai,Y. and Kawamata, S., 1973)

$$f_r\,(D_1, D_2, \ldots, D_n, \Lambda) \equiv f_r\,(D_i, \Lambda) = 0, \quad (r = 1,2,\ldots,n), \tag{4}$$

where $\{D_i\}$ are the displcement parameters, Λ the load parameter, and n the degrees of freedom. For changes in displacement parameter $\{d_i\}$ and in load λ, by Taylor's expansion, the following equation can be obtained

$$\begin{aligned} f_r\,(D_i^{\circ} + d_i, \Lambda^{\circ} + \lambda) &= f_r\,(D_i^{\circ}, \Lambda^{\circ}) + \left(d_i \frac{\partial}{\partial D_i} + \lambda \frac{\partial}{\partial \Lambda}\right) f_r^{\circ} \\ &+ \frac{1}{2!}\left(d_i \frac{\partial}{\partial D_i} + \lambda \frac{\partial}{\partial \Lambda}\right)^2 f_r^{\circ} + \cdots = 0\,; \\ &(r = 1,2,\ldots,n)\ . \end{aligned} \tag{5}$$

The change in displacements and load (d_i, λ) can be indicaed as functions of a parameter t with conditions $d_i(0) = 0; \lambda(0) = 0$,

$$d_i(t) = d_i' t + \frac{1}{2!} d_i'' t^2 + \cdots , \tag{6}$$

$$\lambda(t) = \lambda' t + \frac{1}{2!} \lambda'' t^2 + \cdots . \tag{7}$$

Substituding (6) and (7) into (5), and considering that equation (5) should be tenable for arbitrary t, we obtain the perturbation equation

$$\sum_{i=1}^{n} k_{ri} d_i' = f_{r\lambda} \lambda' , \tag{8}$$

$$\sum_{i=1}^{n} k_{ri} d_i'' + 2 \left(\sum_{i=1}^{n} \sum_{j=1}^{n} k_{rij} d_i' d_j' + \sum_{i=1}^{n} k_{ri\lambda} d_i' \lambda' - f_{r\lambda\lambda} \lambda'^2 \right) = f_{r\lambda} \lambda'' . \tag{9}$$

where,

$$k_{ri} = \frac{\partial f_r}{\partial D_i}; \quad k_{rij} = \frac{1}{2} \frac{\partial^2 f_r}{\partial D_i \partial D_j}; \quad k_{ri\lambda} = \frac{\partial^2 f_r}{\partial D_i \partial \Lambda};$$

$$f_{r\lambda} = -\frac{\partial f_r}{\partial \Lambda}; \quad f_{r\lambda\lambda} = -\frac{1}{2} \frac{\partial^2 f_r}{\partial \Lambda^2} .$$

Equation (8) can be indicated in matrix form

$$\mathbf{K d}' = \mathbf{f} \lambda' , \tag{10}$$

where $\mathbf{K}$ is the tangential stiffness matrix, $\mathbf{d}$ is vector of change in displacements and $\mathbf{f}$ is the load model vector. By load incremental method, taking $t = \lambda$, $\lambda' = d\lambda/dt = 1$, $\lambda'' = 0$, omitting the high order, equation(10) takes the form

$$\mathbf{K d} = \lambda \mathbf{f} , \tag{11}$$

which is the basic equation of load incremental method for finding the load deformation curve and the buckling analysis. Equation (9) is used to analyze the buckling-type.

2.3 Objective Function And Mathematical Model

For a space frame with a certain configuration and subjected to a certain load system, the buckling-load is sensitive to its erection shape, or to its nodal coordinates. So, it is very important to find its optimum shape (nodal coordinates) with maximum buckling-load. The objective function to be maximized is the load-parameter Λ .

By load incremental method, the iteration is done step by step with the change of load-parameter. In the iteration, the buckling point is reached if $\det \mathbf{K} = 0$. For an iteration reaching the buckling point from step 0 to step T, we have

$$\sum_{s=0}^{T} (\mathbf{K}^s \mathbf{d}^s - \lambda^s \mathbf{f}) = 0 \,. \tag{12}$$

Take

$$\sum_{s=0}^{T} \lambda^s = \Lambda \tag{13}$$

as the objective function to be maximized, equation (12) can be rewritten as

$$\sum_{s=0}^{T} \mathbf{K}^s \mathbf{d}^s - \Lambda \mathbf{f} = 0 \,. \tag{14}$$

Hence, the objective function Λ takes the form

$$\Lambda = \mathbf{g} \sum_{s=0}^{T} \mathbf{K}^s \mathbf{d}^s \quad , \tag{15}$$

where $\mathbf{g} = \frac{1}{n}\left(f_1^{-1},\ f_2^{-1},\ \ldots,\ f_n^{-1}\right)$. Then the problem can be expressed as

$$\begin{cases} \text{Maximize} & \Lambda = \mathbf{g} \sum_{s=0}^{T} \mathbf{K}^s \mathbf{d}^s \\ \text{Subject to constraints} & \\ & \Psi(\mathbf{X}) = 0 \\ & \psi_i(\mathbf{X}) \le 0 \,. \end{cases}$$

2.4 The Iteration

In general, the geometrical and mechanical constraints can be divided into active constraints (equalities) and inactive constraints (inequalities).

Some of the geometrical constraints may be given by the architectural requirements or conditions for some specific purposes. Inactive constraints should be adopted to meet the architectural requirements, and to control the locations of the joints not to be too high or too low, or to make it a dome-like one.

The mechanical constraints are always given according to the mechanical principles. If prestresses are introduced in the structure, however the shape be changed by the optimization, the internal forces introduced by prestressing must be in equilibrium without loads, which is the active constraints (ref. Chen,P.S., Kawaguchi,M.,1993).

The geommetrical constraint equations, linear or nonlinear, make a constraint space for the nodal cartesian-coordinates of the structure. The nodal cartesian-coordinates can be defined by a base of the constraint space, or in other words, the nodal cartesian-coordinates $\mathbf{X} = (x_1, x_2, \ldots, n)$ can be indicated by a generalized coordinate system $\mathbf{q} = (q_1, q_2, \ldots, q_m) \quad n \geq m$, which is the independent variable of the optimization.

$$\left.\begin{array}{l} x_1 = \mathcal{X}_1(q_1, q_2, \ldots, q_m) \\ x_2 = \mathcal{X}_2(q_1, q_2, \ldots, q_m) \\ \cdots\cdots\cdots\cdots\cdots\cdots \\ x_n = \mathcal{X}_n(q_1, q_2, \ldots, q_m) \end{array}\right\} \quad ; \quad (n \geq m) \ . \tag{16}$$

Probably, transformation (16) have to be expressed in implicit function, but the Jacobian matrix of the transformation can be obtained

$$\mathbf{J} = \begin{pmatrix} \partial x_1/\partial q_1 & \cdots & \partial x_n/\partial q_1 \\ \vdots & & \vdots \\ \partial x_1/\partial q_m & \cdots & \partial x_n/\partial q_m \end{pmatrix} \ . \tag{17}$$

The incremental direction of the objective function can be obtained by its derivative with respect to the nodal coordinates.

$$\dot{A}_i = \dot{\mathbf{g}}_i \sum_{s=0}^{T} \mathbf{K}^s \mathbf{d}^s + \mathbf{g} \sum_{s=0}^{T} \dot{\mathbf{K}}_i^s \mathbf{d}^s \quad . \tag{18}$$

where $\dot{A}_i = \partial A / \partial x_i$. Generalized incremental direction respecting the generalized coordinates $\mathbf{q}$ takes form

$$\begin{pmatrix} A'_1 \\ A'_2 \\ \vdots \\ A'_m \end{pmatrix} = \mathbf{J} \begin{pmatrix} \dot{A}_1 \\ \dot{A}_2 \\ \vdots \\ \dot{A}_n \end{pmatrix} \quad , \tag{19}$$

where $A'_i = \partial A / \partial q_i$. The direction moving to the next step of the iteration takes the form

$$\forall\, i \quad \begin{cases} \Gamma_i = 0 \quad , & \text{if } \ A'_i \leq 0 \\ \Gamma_i = A'_i \quad , & \text{otherwise} \quad . \end{cases}$$

If $\mathbf{\Gamma} = \mathbf{0}$ then stop the calculation, or else change the coordinates, and continue the calculation,

$$\{q_i\}^{T+1} = \{q_i\}^T + \alpha \{\Gamma_i\}^T \quad ; \qquad (i = 1, 2, \ldots, m) \quad , \tag{20}$$

where α is the step-length which is defined by

$$0 \leq \alpha \leq \min \left\{ \Psi_j \left(\mathbf{q}^T \right) / \nabla^{\mathrm{T}} \Psi_j^T A_i'^T \ \mid \nabla^{\mathrm{T}} \Psi_j^T A_i'^T \geq 0 \right\} . \tag{21}$$

Conclusion

The present analysis is the first step of our research, and there are many important topics need deeper researches, for example, the multicriteria optimization for maximum backing-load and minimum volume with sensitivity analyses etc..

The method presented is very convenient, and is proposed as a useful aid in design and design automation.

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