For IASS/APCS 2006

MAXIMUM BUCKLING LOAD OF LATTICE SPACE FRAMES AND TOPICS ON IMPERFECTION SENSITIVITY

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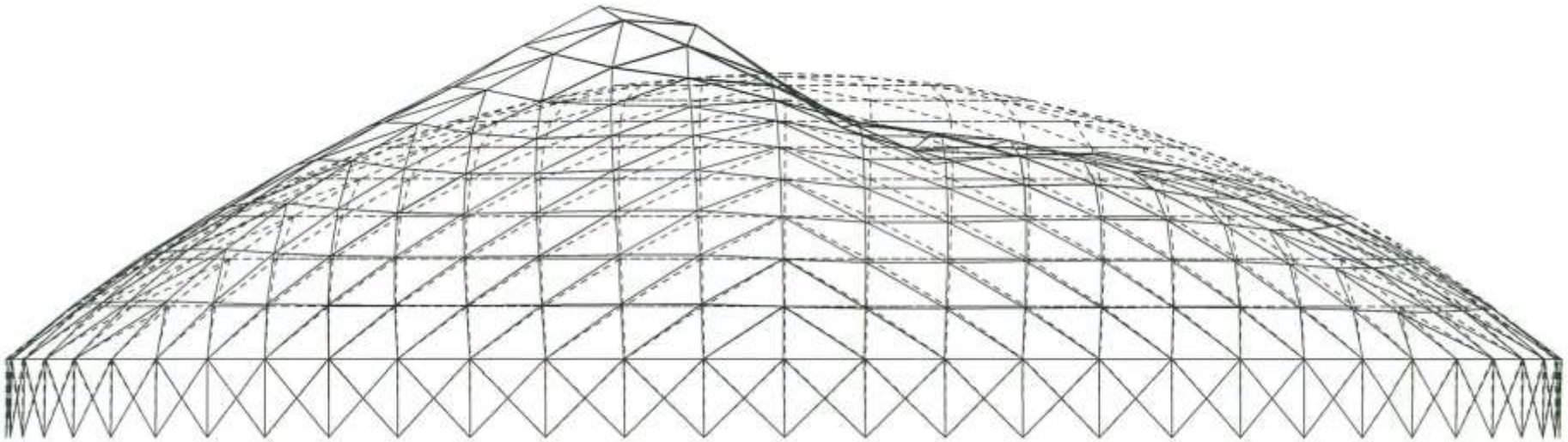
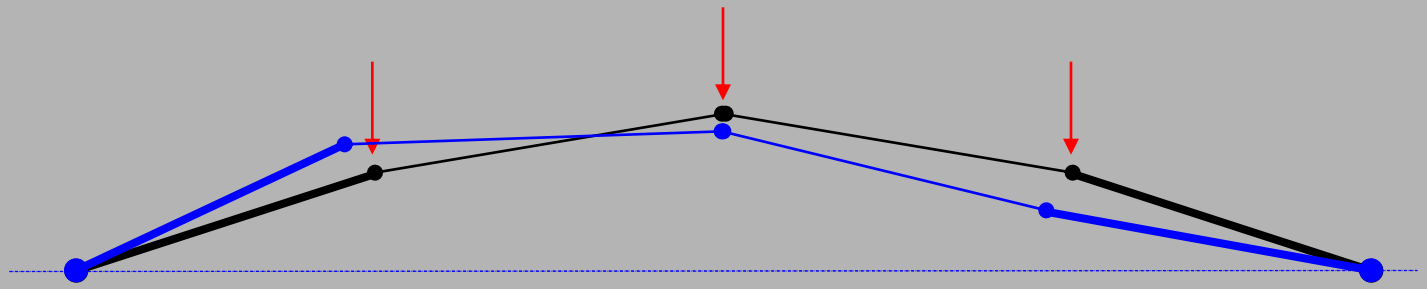
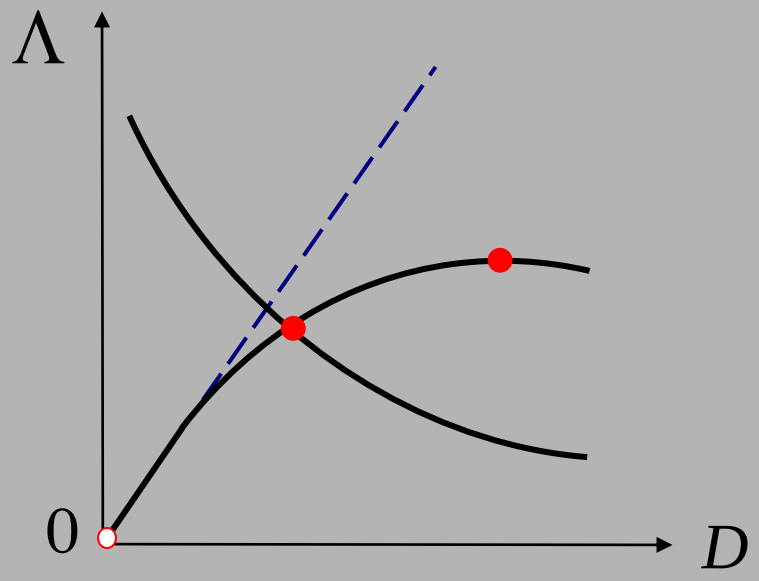
Concept of Buckling



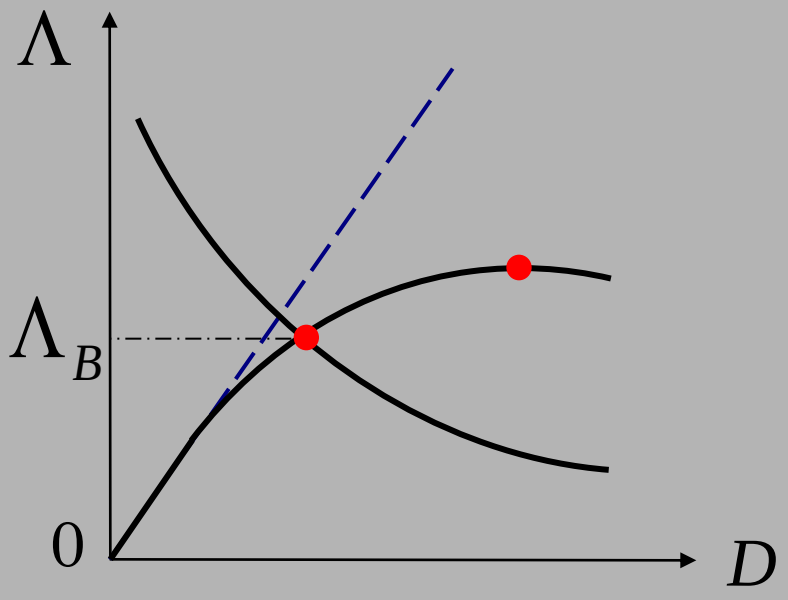
Concept of Buckling



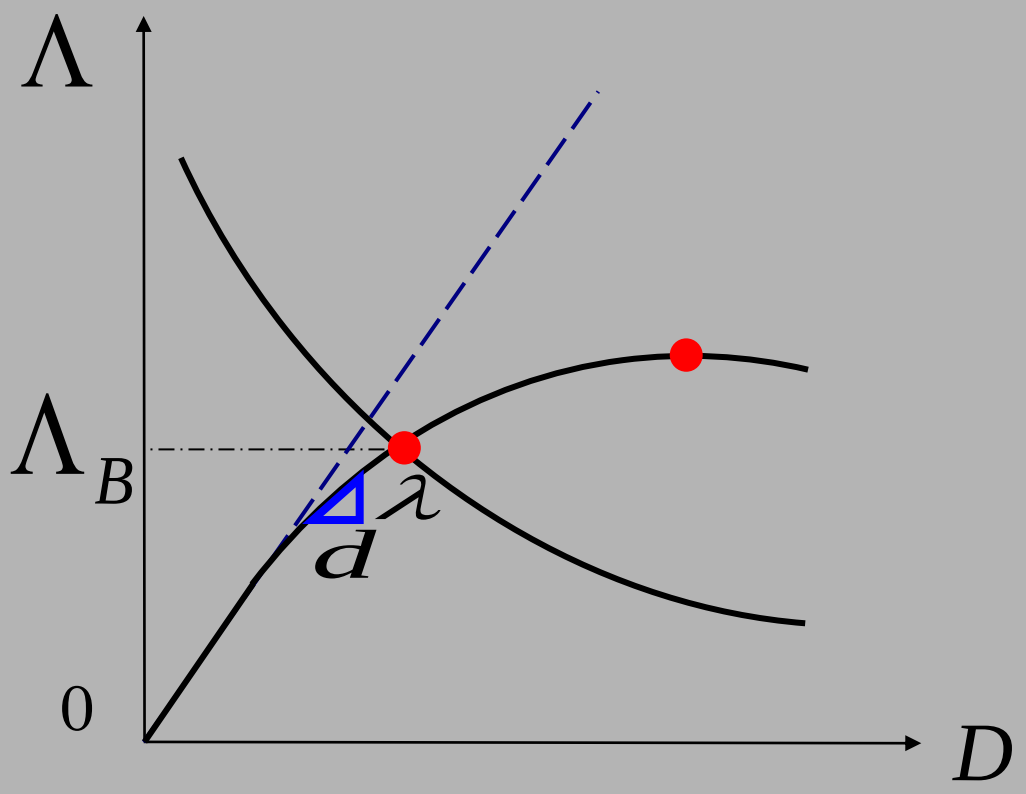


The Optimization for Maximum Buckling Load

$$\begin{cases} \text{Maxmize } \Lambda_B \\ \text{Subject to } \mathbf{h}(\mathbf{X},\mathbf{U}) = \mathbf{0} \\ \qquad\qquad\quad \mathbf{g}(\mathbf{X},\mathbf{U}) \le \mathbf{0} \end{cases}$$



To Find the Bucking Load



$$\frac{K}{d}\lambda$$

$$\mathbf{Kd} = \lambda \mathbf{f}$$

$$\Lambda_B = \sum_{u=1}^{T} \lambda^u$$

Bucking Load in Explicit Function

$$\mathbf{Kd} = \lambda \mathbf{f}$$

$$b_i = \begin{cases} 0 & , \ \forall f_i = 0 \\ \frac{1}{\mathrm{N}} f_i^{-1}, & \forall f_i \neq 0 \end{cases}$$

$$\mathbf{b}^{\mathrm{T}}\mathbf{f} = 1$$

$$\lambda^u = \mathbf{b}^{\mathrm{T}}\mathbf{K}^u\mathbf{d}^u$$

Objective Function for Optimization

$$\lambda^u = \mathbf{b}^{\mathsf{T}} \mathbf{K}^u \mathbf{d}^u$$

$$\Lambda_B = \sum_{u=1}^{T} \lambda^u$$

$${}_i\Lambda = {}_i\mathbf{b}^{\mathsf{T}} \sum_{u=1}^{S} {}_i\mathbf{K}^u \; {}_i\mathbf{d}^u \quad ; \quad (i = 1,2,\cdots,l)$$

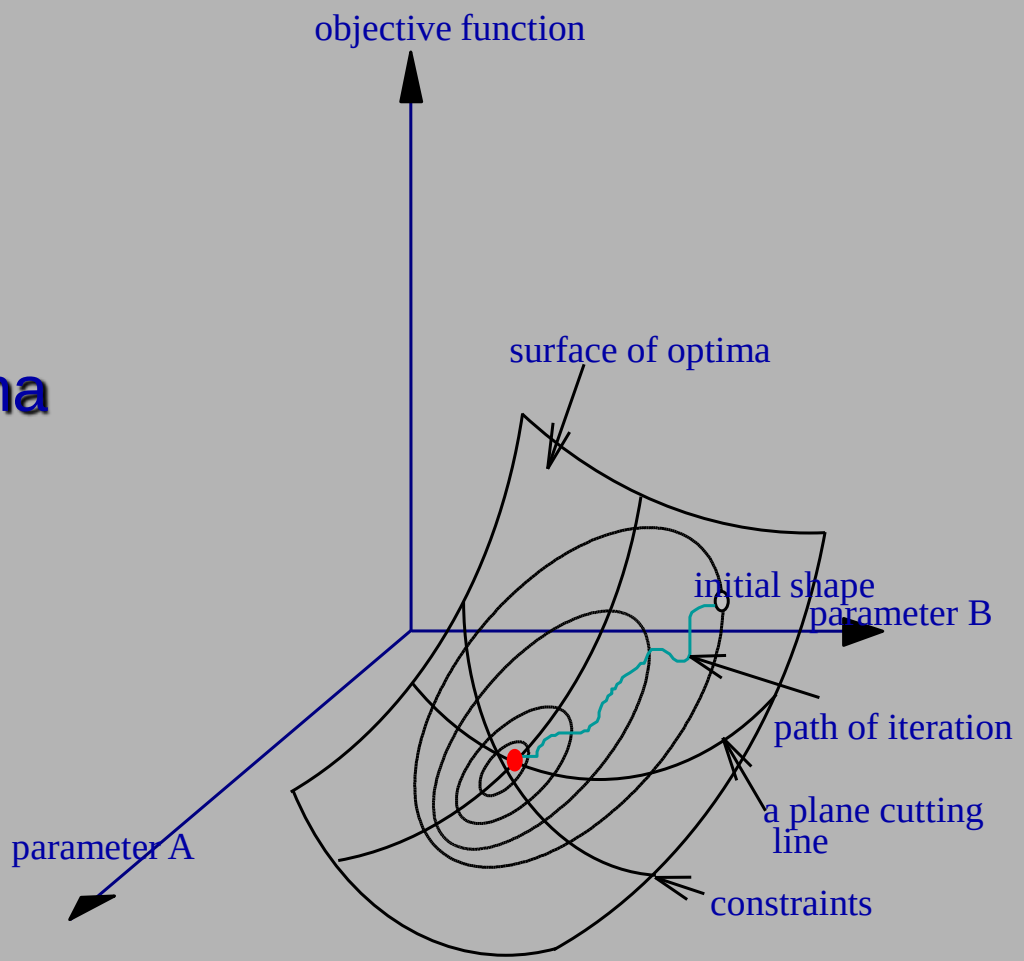
The Mathematical Model

$$\begin{cases} \text{Maximize } F = \sum_{i=1}^{l} w^{i}{}_{i}\Lambda \\ \text{Subject to:} \\ \qquad h_j(\mathbf{X},\mathbf{U}) = 0; \quad (j = 1,\ldots,s) \\ \qquad g_k(\mathbf{X},\mathbf{U}) \le 0, \quad (k = 1,\ldots,t) \end{cases}$$

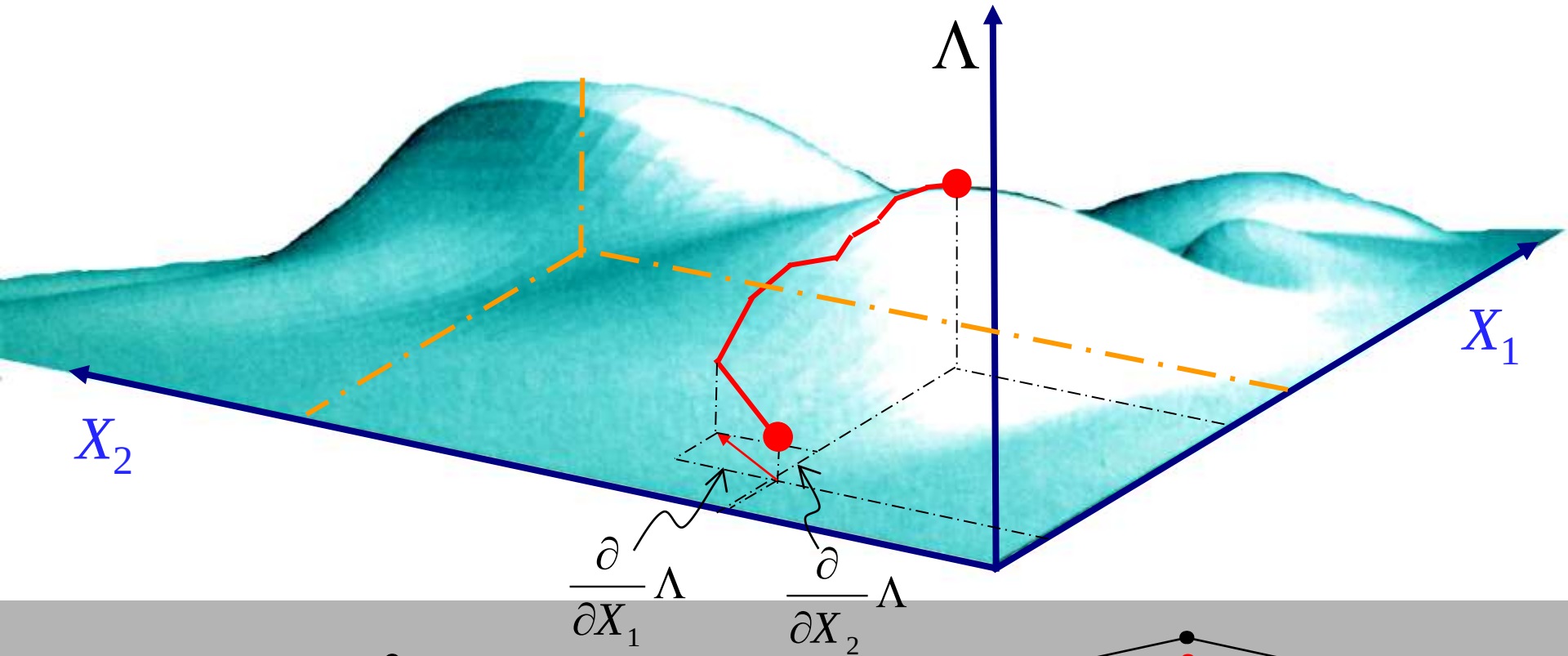
$$ {}_{i}\Lambda = {}_{i}\mathbf{b}^{\mathsf{T}} \sum_{u=1}^{S} {}_{i}\mathbf{K}^{u} \; {}_{i}\mathbf{d}^{u} \quad ; \quad (i = 1,2,\cdots,l)$$

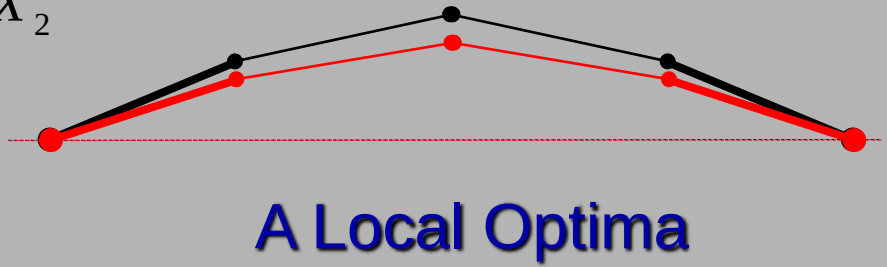
To Find the Optima

The concept of
the surface of optima



A Local Optima, OK ?





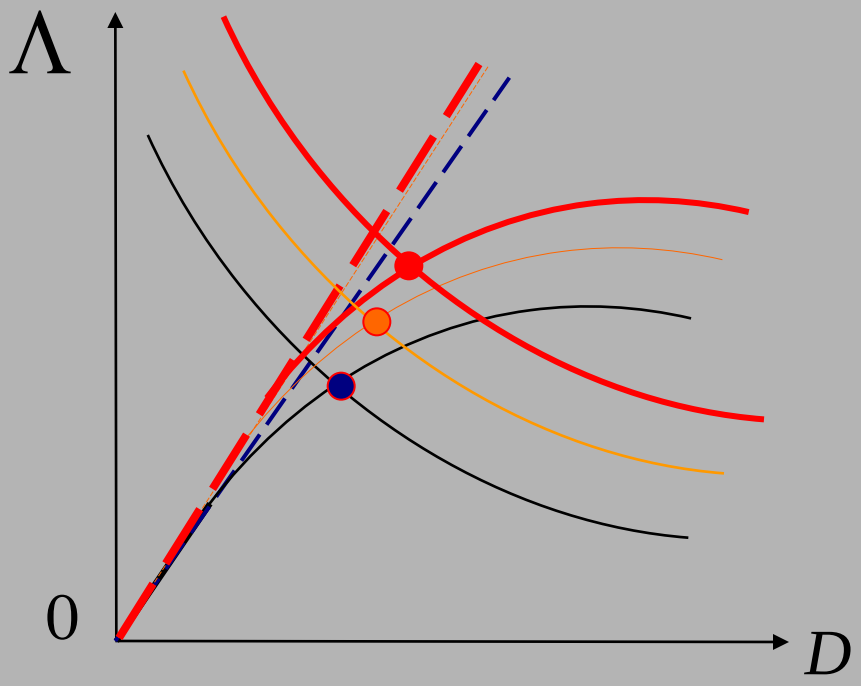
Direction to Change the Shape

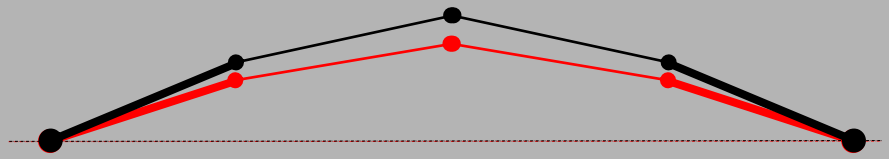
$$\dot{\Lambda}_{,i} = \dot{\mathbf{b}}^{\mathsf{T}}{}_{,i} \sum_{u=1}^{T} \mathbf{K}^{u}\, \mathbf{d}^{u} + \mathbf{b}^{\mathsf{T}} \sum_{u=1}^{T} \dot{\mathbf{K}}^{u}_{,i}\, \mathbf{d}^{u}$$

$$= \Lambda \dot{\mathbf{b}}^{\mathsf{T}}{}_{,i}\, \mathbf{f} + \mathbf{b}^{\mathsf{T}} \sum_{u=1}^{T} \dot{\mathbf{K}}^{u}_{,i}\, \mathbf{d}^{u} \qquad (i = 1,2,\ldots n+m)$$

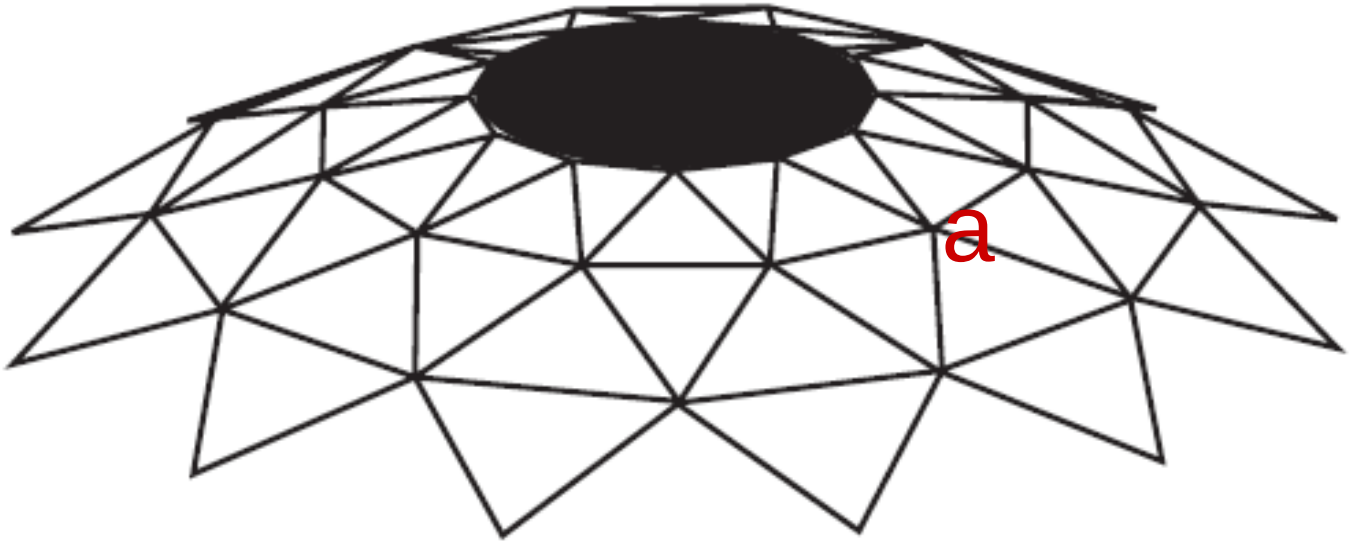
$$\dot{\Lambda}_{,i} = \Lambda \dot{b}_{j,i}\; f_j + b_{j,i}\; \dot{K}_{jl,i}\; d_l$$

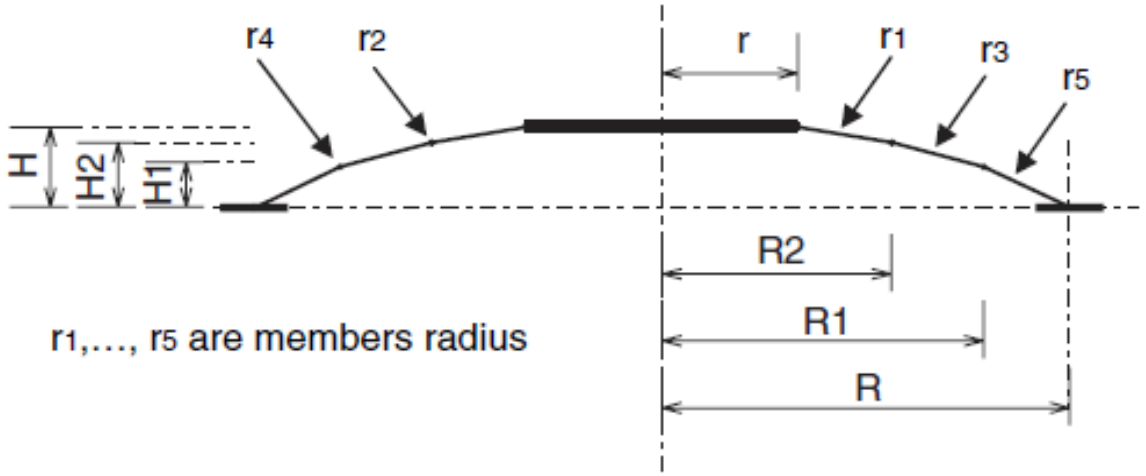
The Process of the Optimization





Numerical Analysis Example





The Analysis Cases

Case 1: Full area loading mode;

Change the shape design parameters;

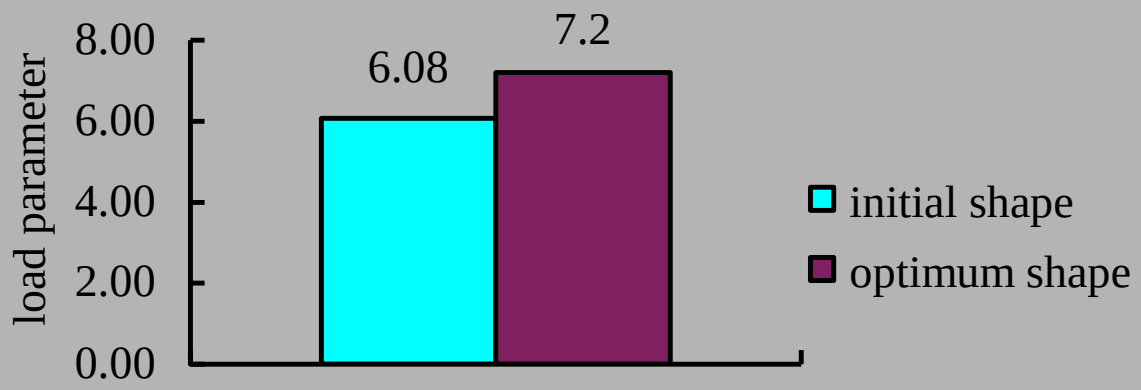
All the member sections are kept constant.

Case 2: Full and Half area loading modes;

Change shape design parameters and the member sections.

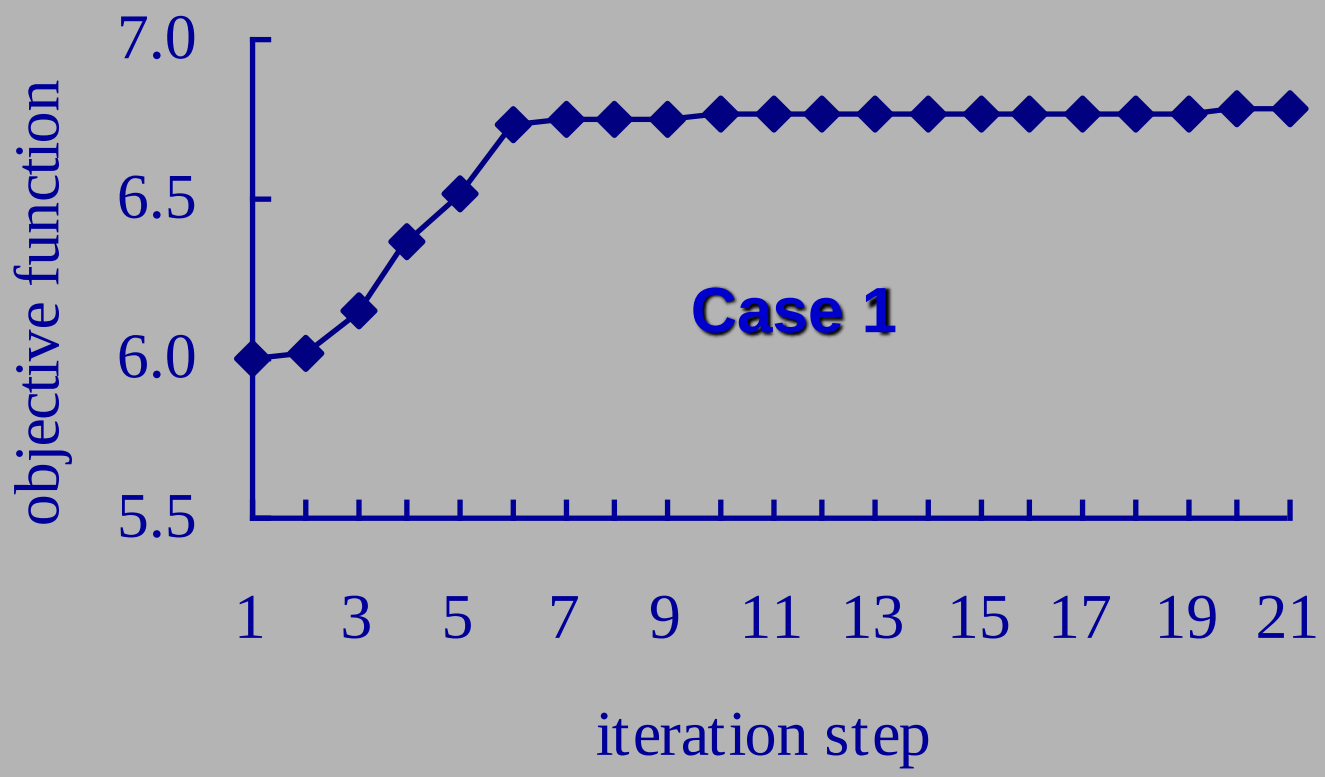
The Result of Case 1

initial shape optimum shape

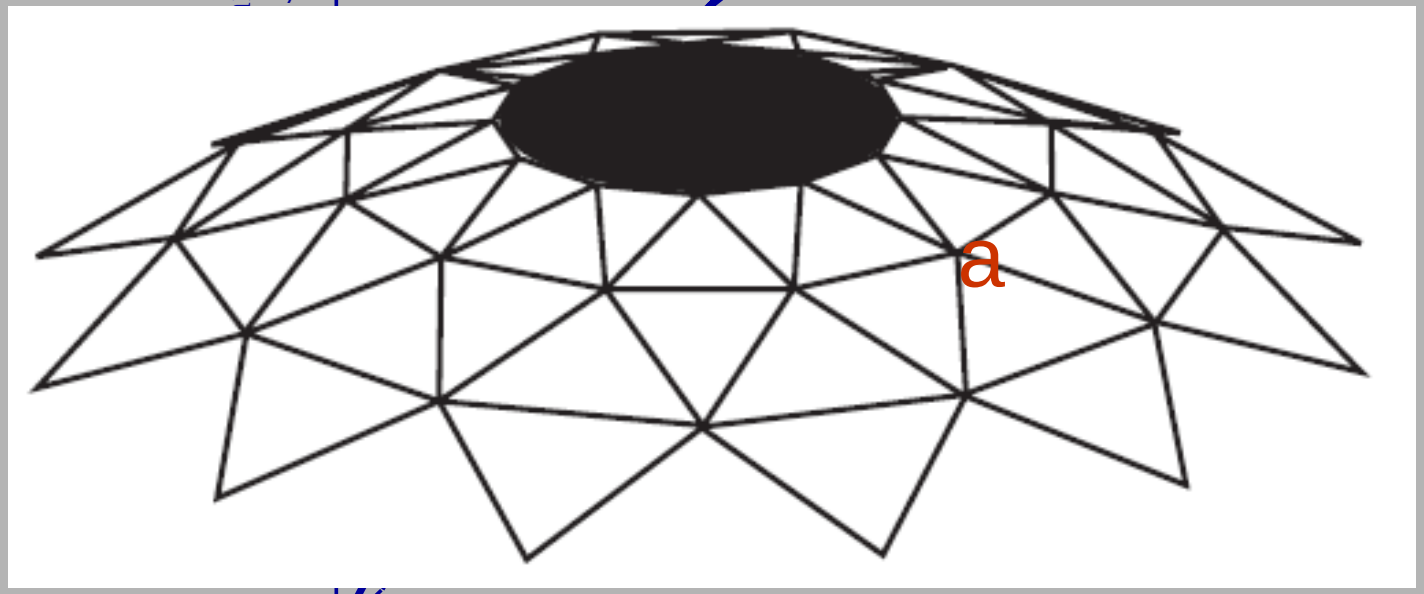


The buckling load parameter increase 18.42%

Convergence of the Numerical Analysis

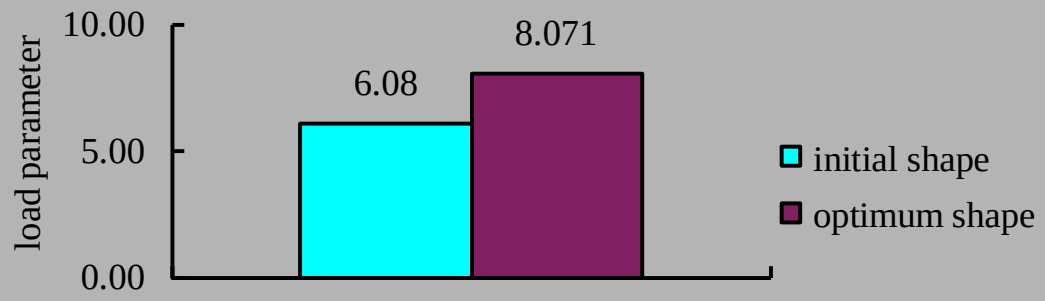


Load-Displacement Curves Case 1

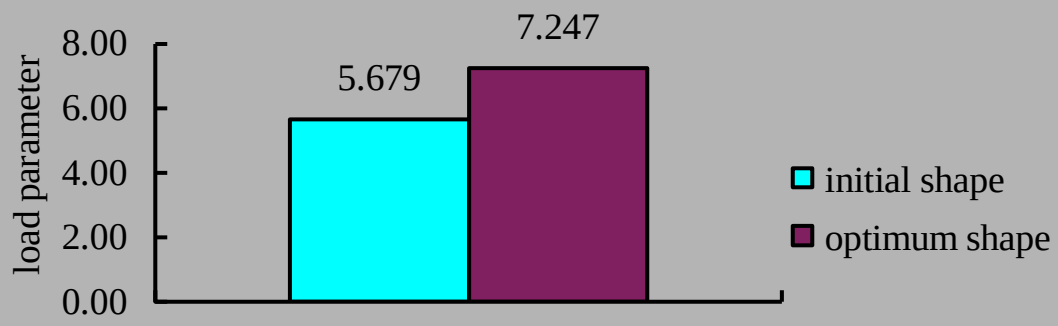


The Load-Displacement Curves at Point **a**

The Result of Case 2

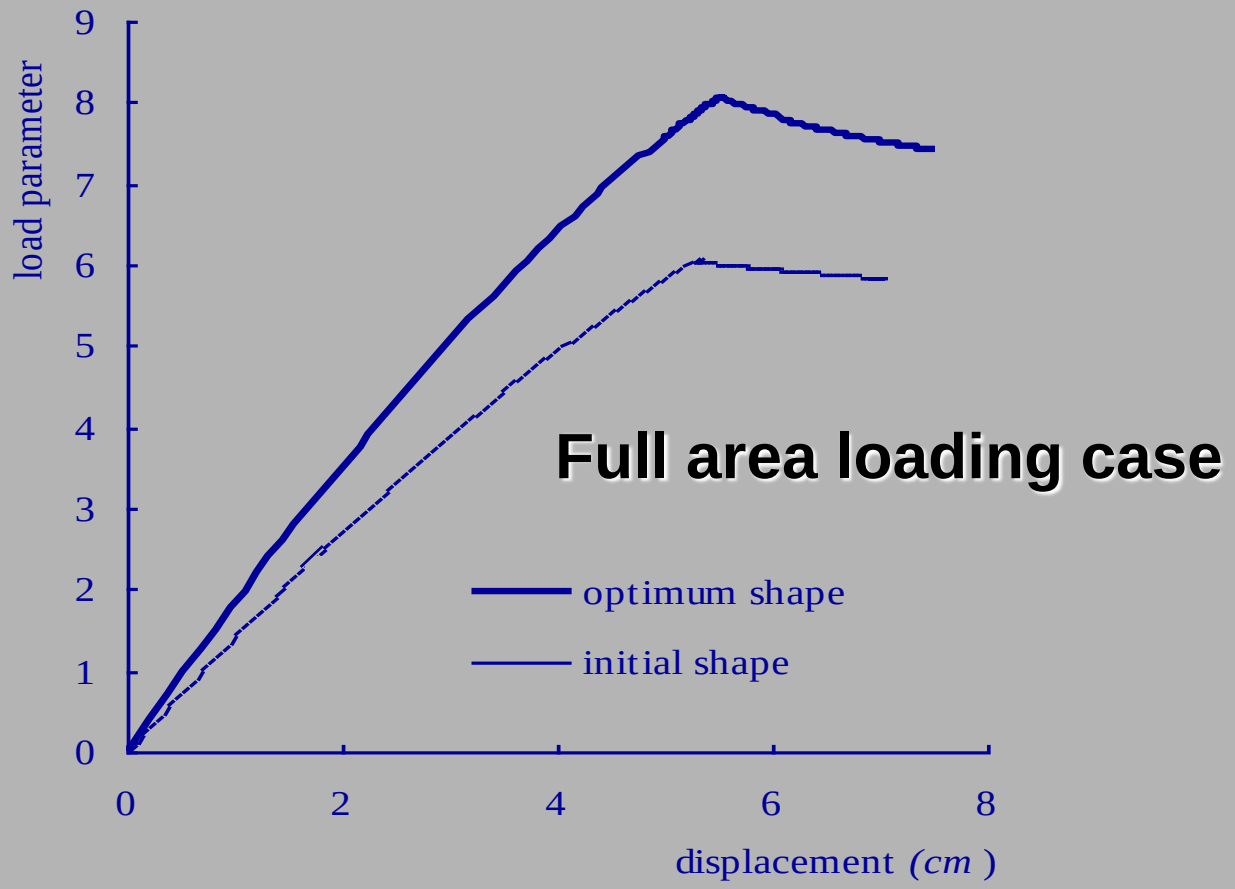


Full area loading case ;
The buckling load parameter increased 32.75%



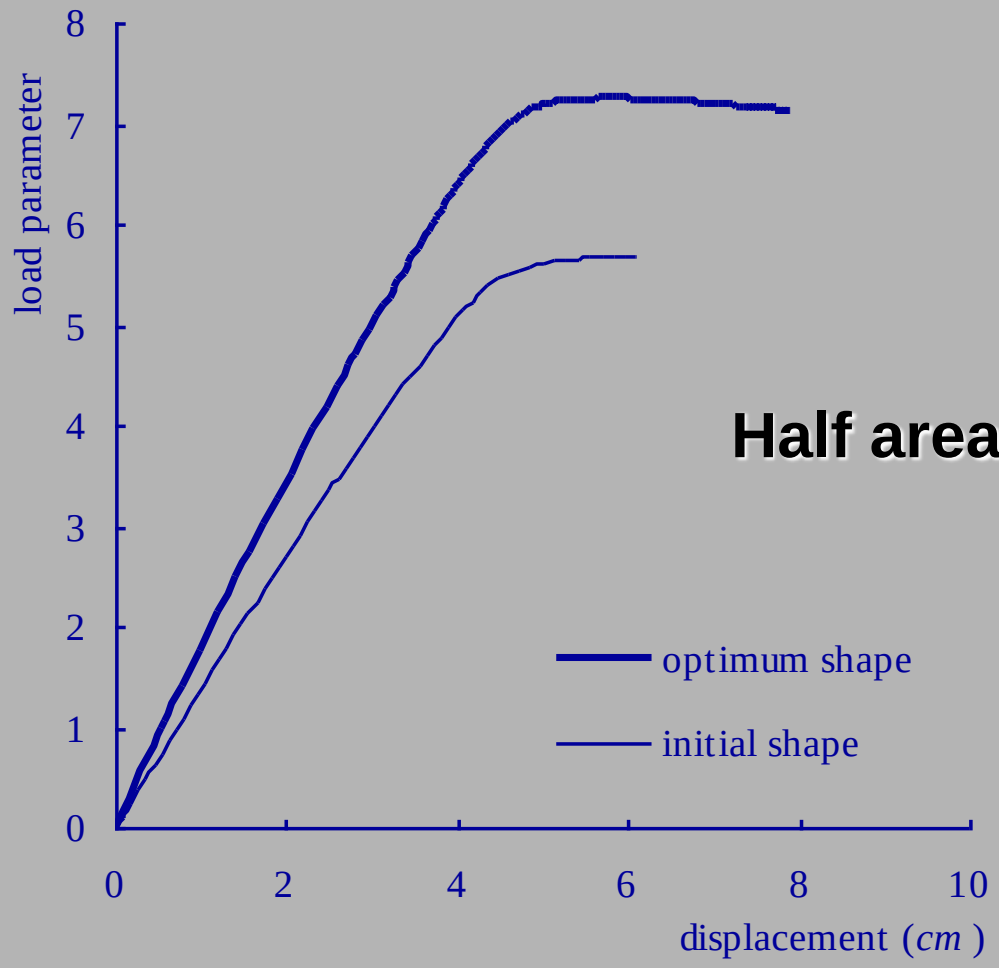
Half area loading case
The buckling load parameter increased 27.61%

Load-Displacement Curves Case 2



The Load-Displacement Curves at Point **a**

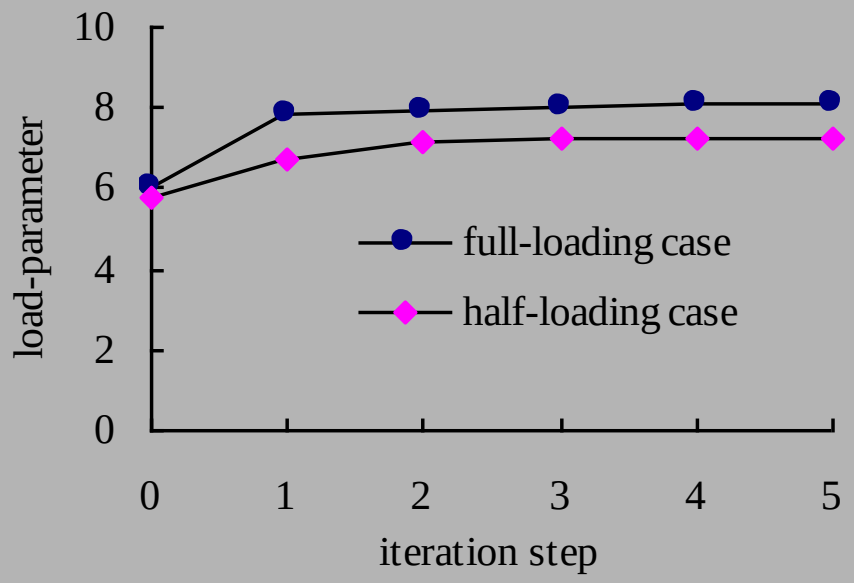
Load-Displacement Curves Case 2



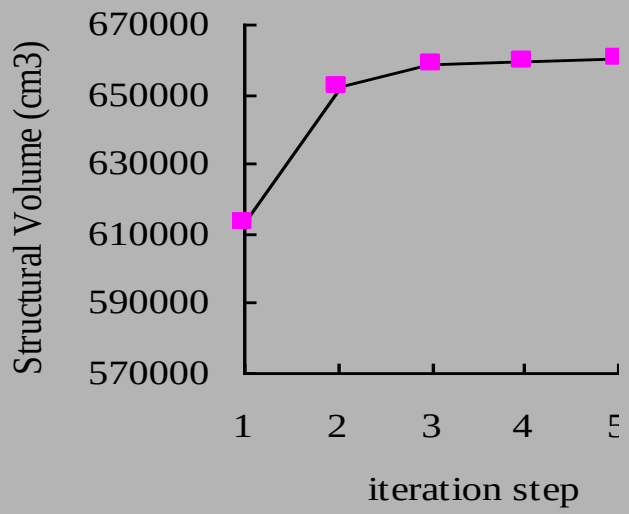
Half area loading case

The Load-Displacement Curves at Point **a**

Convergence of the Numerical Analysis

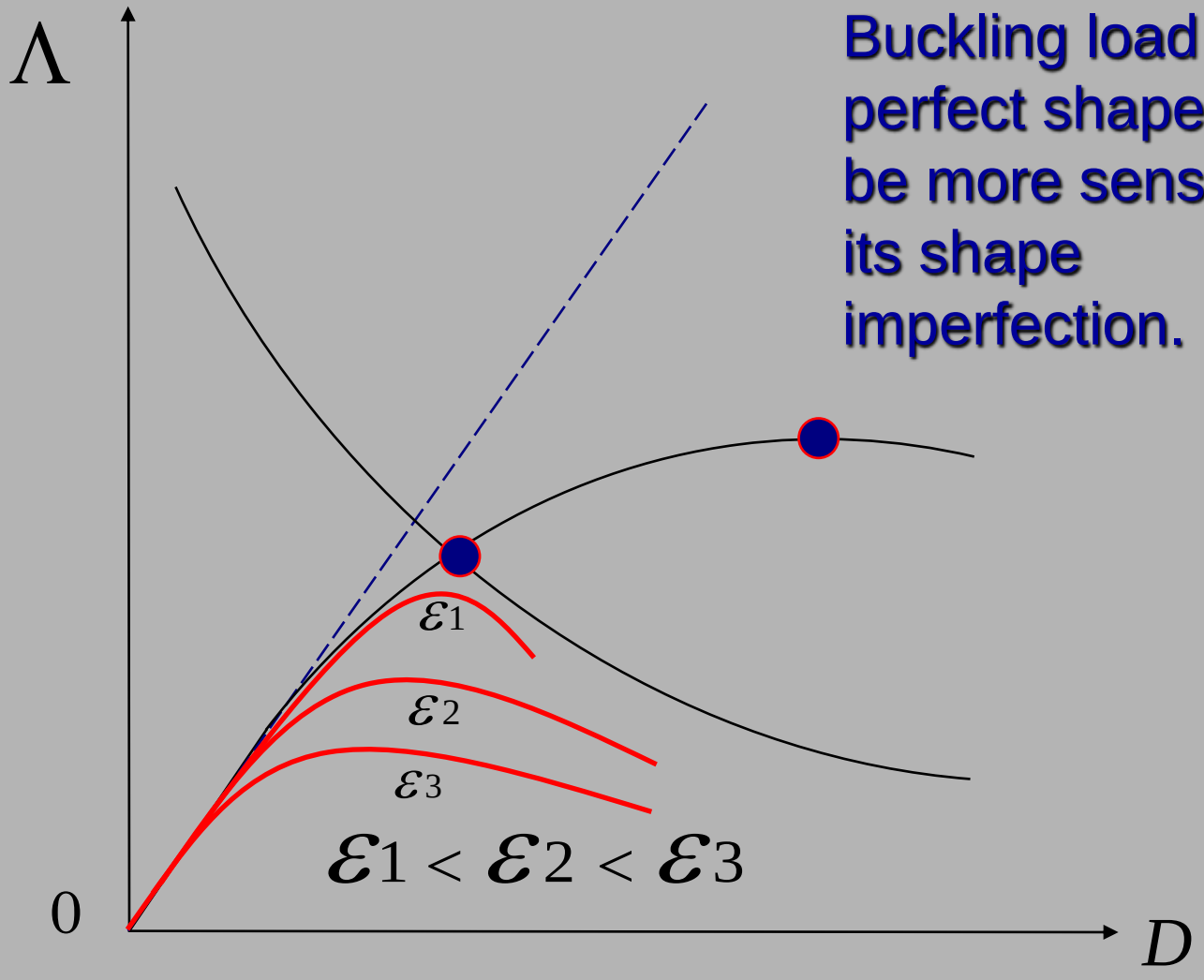


The convergence of the optimization iteration



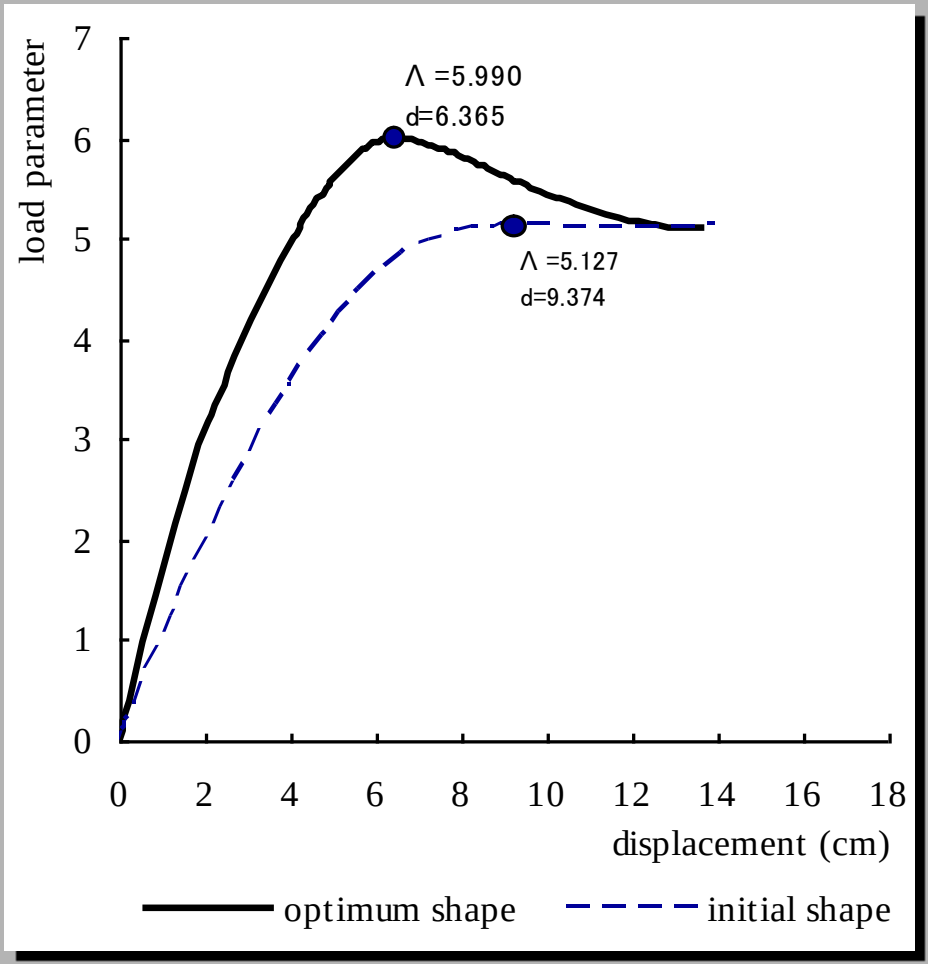
The change of structural volume

Imperfection



Buckling load of a perfect shape may be more sensitive to its shape imperfection. ? ? ?

Imperfection for Case 1

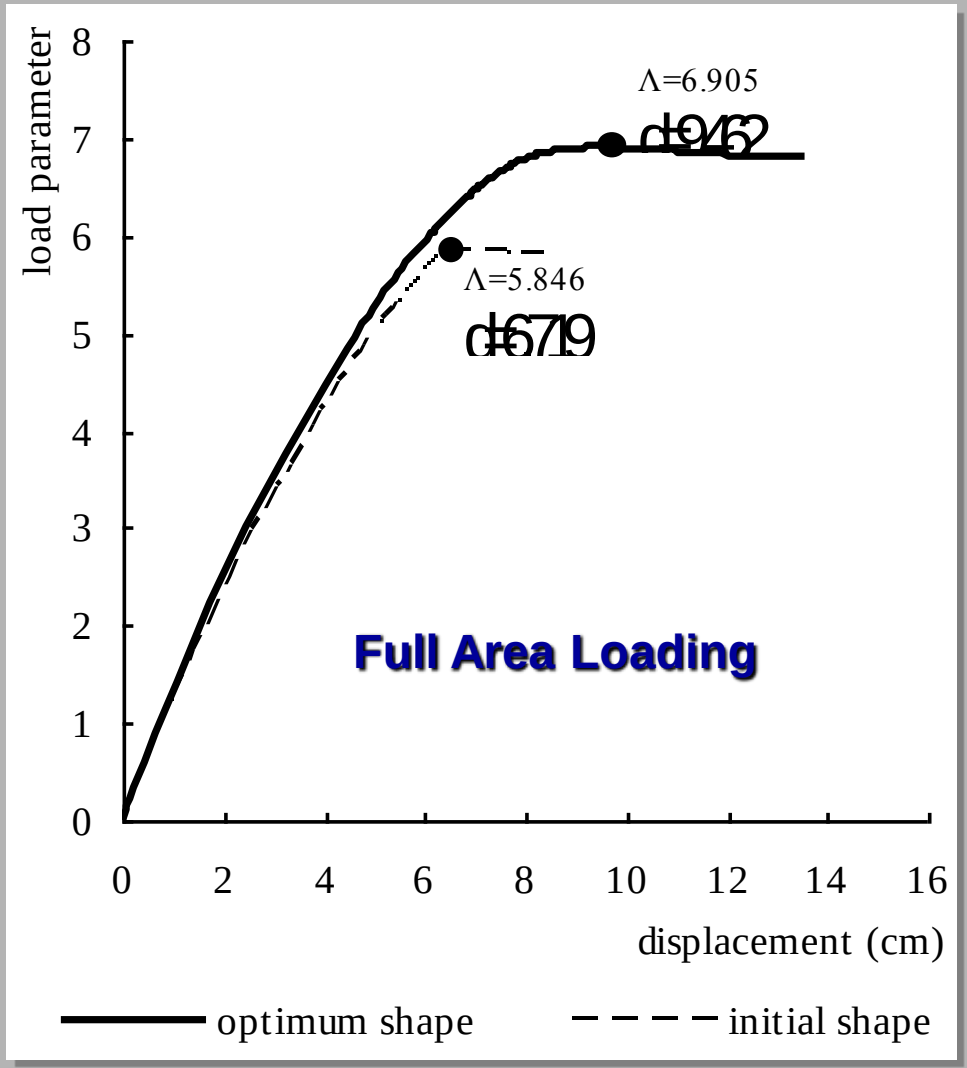


Case 1: Change the shape design parameters only.

Ratio of the maximum imperfection point in second ring is $R1:\varepsilon=1:0.0005$

Full Area Loading

Imperfection for Case 2



Case 2: Change shape design parameters and all of the **member sections**.

Ratio of the maximum imperfection point in second ring is $R1:\varepsilon=1:0.0005$

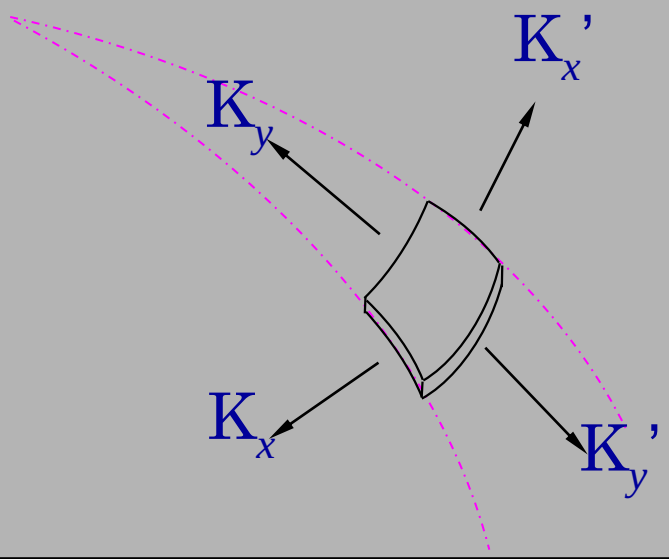
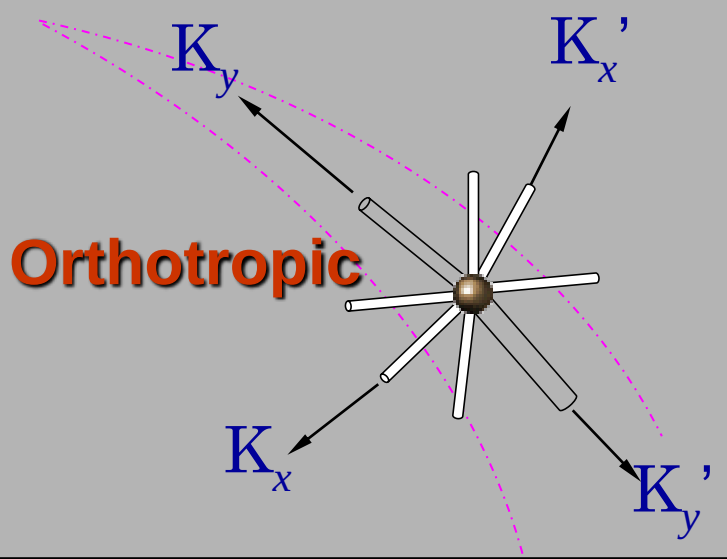
The optimum shape with maximum buckling load may not be sensitive to its shape imperfection.

What is the reason ?

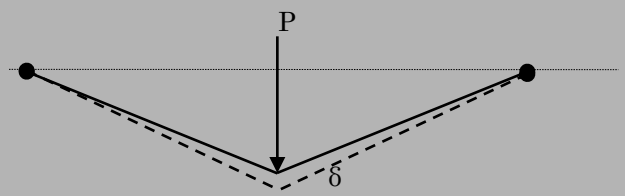
initial shape optimum shape

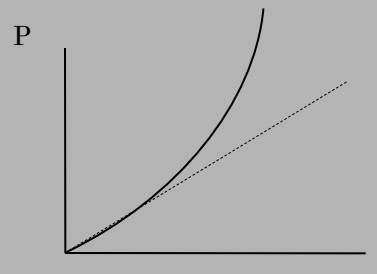
Changes in member sections

Difference Between Shells and Space Frames

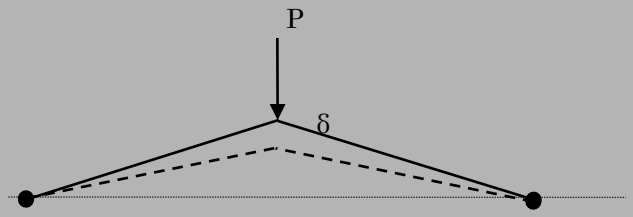
Shells	Space frames
	
1. Built by thin continuum in curvature	1. Built by discrete members
2. The stiffness K_x and K_y are continue respectively and related to each other.	2. The stiffness K_x and K_y are discontinue respectively and independent with each other.
3. Changes in thickness lead to the changes in bending stiffness.	3. Changes in member sections cause slight changes in global bending stiffness.

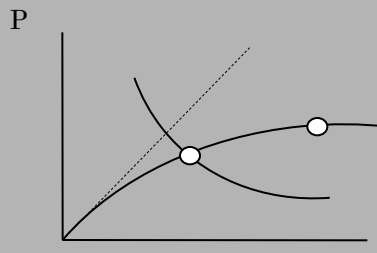
Geometrical Influence on Buckling Behaviors





No stiffness reduction → Do not Buckle

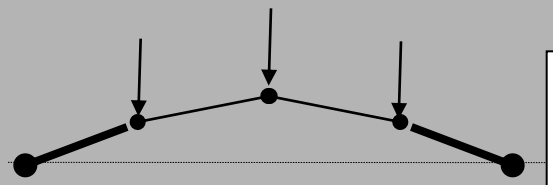




Stiffness reduction:
Limit point
Bifurcation point
→ Buckle

Shape and Stiffness → Buckling ?

Influence on Shape and Stiffness



Stiffness Change by:

- Geometrical Changes
  - Shape & thin Isotropic thickness → **Shell: imperfection Sensitive**
  - Shape & Isotropic discontinue member sections → **Space frame: May not Sensitive to imperfection**
- Mechanical and Material Changes
  - Isotropic → **Imperfection Sensitive**
  - orthotropic → **May not Sensitive to imperfection**

An Inference on the Imperfection Sensitivity

For a lattice space frame as well as a shell with orthotropic material, the optimum shape with maximum buckling load may not be sensitive to its geometrical imperfection.

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