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Optimization for Maximum Buckling Load of a Lattice Space Frame With Nonlinear Sensitivity Analysis

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ABSTRACT: The paper presents an optimization analysis method to find the optimum shape of a lattice space frame with maximum buckling load. The buckling load of the lattice space frame is determined by a geometrically nonlinear analysis. The mathematic analysis of the optimization is based on Generalised Reduced Gradient method (GRG) [1], in which the slope of the objective function (the load parameter at buckling point) with respect to the design parameters is required. In order to determine the slope, the authors introduced a so-called Stiffness Sensitivity Tensor in the nonlinear structural analysis, such that the sensitivity analysis can be carried out simultaneously with the incremental iteration of the geometrically nonlinear structural analysis. A single layer lattice dome is taken as a numerical example, and the results show that the buckling load parameter for full area loading case increases 32.75% compared to that for the initial shape.

Key Words: Shape optimization, maximum buckling load, sensitivity analysis, Stiffness Sensitivity Tensor, nonlinear analysis, lattice space frame, lattice dome.

1. INTRODUCTION

Stability discussion plays an important role in structural design of large span spatial structures, especially for the single layer lattice dome structures. Ramm and Reilinger [2, 3] have presented optimization methods for shells. However, there are few researches focusing on the optimum shape of a lattice space frame with maximum buckling load. In the past years, the authors have researched such a theme and issued parts of their works [4, 5]. In the present paper, the authors will present an optimization method for maximum buckling load of a lattice space frame.

For most optimization methods with mathematic treatments, sensitivity of the objective function with respect to the design parameters is required. But the sensitivity analysis based on a nonlinear structural analysis is known as a difficult problem, especially when material nonlinear behaviors have to be taken into account. In the present paper, the authors define a so-called Stiffness Sensitivity Tensor, by which the sensitivity of the objective function with respect to the design parameters can be determined simultaneously with the iteration of geometrically nonlinear structural analysis. Therefore, the derivative of buckling load parameter with respect to design parameters can be obtained easily. GRG is adopted for the optimization in the present paper, in which the incremental direction of the objective function is determined by the derivative of the objective function with respect to the design parameters. The objective function in the present optimization is the buckling load parameter of a lattice space frame, which is determined by a geometrically nonlinear structural analysis.

2. FORMULATION

2.1. The basic theory

In general, a nonlinear structural analysis is based on the incremental equation shown below.

$$\mathbf{Kd} = \lambda\mathbf{f} \tag{1}$$

where $\mathbf{K}$ is the tangential stiffness matrix, $\mathbf{d}$ the vector of changes in displacements, $\mathbf{f}$ the load mode vector, and λ the change in load parameter. Many numerical techniques, such as Riks-Method, load incremental and displacement incremental methods are developed to find the equilibrium path (load-displacement curve) [6–8], and the buckling point is obtained as a limit point and/or a bifurcation point in the load-displacement curve.

For a common analysis procedure, the load mode vector keeps constant, and its coefficient, the load parameter, is increased step by step by the iteration.

For the convenience of the mathematic analysis, we make the load parameter as an explicitly objective function. As the first step, we define a vector $\mathbf{b}$ by the following equations.

$$\mathbf{b}^{\mathrm{T}}\mathbf{f} = 1 \tag{2a}$$

$$b_i = \begin{cases} 0, & \forall f_i = 0 \\ \dfrac{1}{\mathrm{N}} f_i^{-1}, & \forall f_i \neq 0 \end{cases} \tag{2b}$$

where $\mathbf{f} = \{f_i\}$ and N is the number of nonzero elements of $\mathbf{f}$. Then, an explicit function of the load parameter at incremental step u can be obtained as the following equation.

$$\lambda^u = \mathbf{b}^{\mathrm{T}}\mathbf{K}^u\mathbf{d}^u \tag{3}$$

If the buckling point is found at step T of the nonlinear structural analysis, the total buckling load parameter Λ can be obtained as:

$$\Lambda = \mathbf{b}^{\mathrm{T}}\sum_{u=1}^{T}\mathbf{K}^u\mathbf{d}^u \tag{4}$$

Then, the optimization for maximum buckling load of a lattice space frame can be formulated as below.

$$\begin{cases} \text{Maximize} \quad \Lambda = \mathbf{b}^{\mathrm{T}}\sum_{u=1}^{T}\mathbf{K}^u\mathbf{d}^u & (5) \\ \text{Subject to :} & \\ \quad h_j(\mathbf{X},\mathbf{U}) = 0\ ; \quad (j=1,\ldots,s) & (6) \\ \quad g_k(\mathbf{X},\mathbf{U}) \leq 0\ , \quad (k=1,\ldots,t) & (7) \end{cases}$$

where $\mathbf{X}$ is the joint coordinate vector with n elements and $\mathbf{U}$ the other design parameters vector with m elements. Equation (6) indicates the equality constraints, Equation (7) the inequality constraints, and these conditions make a constraint space S.

$$S = \begin{cases} \mathbf{X} \subset \Re^n \\ \mathbf{U} \subset \Re^m \end{cases}$$

$$\left| \begin{array}{ll} g_i(\mathbf{X},\mathbf{U}) \leq 0, & h_j(\mathbf{X},\mathbf{U}) = 0 \\ i = 1,2,\ldots t, & j = 1,2,\ldots s < n+m \end{array} \right\} \tag{8}$$

Then, the process of the optimization is to search a point of the design parameters $(\mathbf{X},\ \mathbf{U})$, $\mathbf{X}\in S$ and $\mathbf{U}\in S$, at which the total buckling load parameter Λ reaches its maximum.

2.2. Formulation for multi-loading modes

In fact, a structure should be able to support multi-loading cases. On the other hands, the optimum shape found by the present method depends on the loading mode $\mathbf{f}$. Therefore, the authors present an optimization for multi-loading modes by introducing the weighting function method.

For every loading mode ${}_i\mathbf{f}$ $(i=1,2,\ldots,l)$, we can define a vector ${}_i\mathbf{b}$ $(i=1,2,\ldots,l)$ by Equations (2a, 2b) and the load parameter by Equation (3). Then, the total load parameters ${}_i\Lambda$ $(i=1,2,\ldots,l)$ can be obtained by

$${}_i\Lambda = {}_i\mathbf{b}^T\sum_{u=1}^{T}{}_i\mathbf{K}^u\ {}_i\mathbf{d}^u; \quad (i = 1,2,\ldots,l) \tag{9}$$

Then the objective function is in form

$$F = \sum_{i=1}^{l} w^i\ {}_i\Lambda \tag{10}$$

where $w^1,\ldots,w^l$, $\sum_i w^i = 1$ are weighting factors which are determined by the designers. Then the optimization for multi-loading cases can be formulated as:

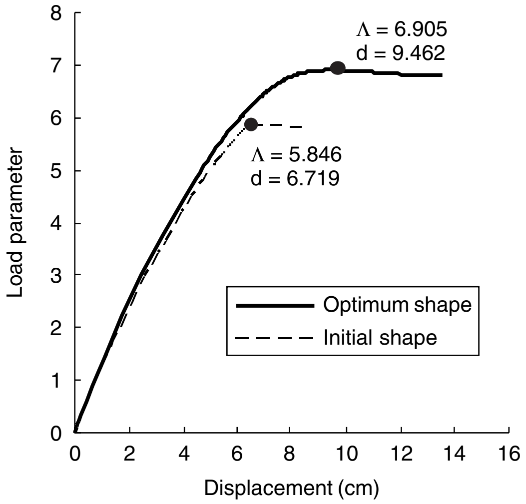

Figure 15. The load-displacement curves of point **a** Shape imperfection for case 2; Full area loading case; Point **a** is shown in Fig. 3(b).

maximum buckling load with member section dimensions as design parameters is more efficient, and can provide more information for the structural working design. Hence, an optimum shape with maximum buckling load may not be more sensitive to its shape imperfection.

5. CONCLUSION

By mathematical analysis and numerical examples, the authors demonstrate that the present method is an effective method for the shape optimization of maximum buckling load. However, this method can find the optima only in the vicinity of the initial design parameters, but it may not be efficient to find the global optima in constraint space. The present optimization can determine the member sections automatically and out put more information for the structural working design. Authors deduct that optimization for maximum buckling load with member section dimensions as design parameters is more efficient than that with geometrical shape-parameters only.

The numerical example demonstrated that an optimum shape with maximum buckling load may not be sensitive to its shape imperfection, especially when member section dimensions are taken as design parameters. Nevertheless, a mathematic theory of imperfection-sensitivity analysis for the present optimization is remained. Instead of the imperfection sensitivity analysis, the authors promoted a method of comparing the nonlinear behaviors between the imperfection shapes of the optimum and initial ones.

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