Overall stiffness evaluation and shape optimisation of a tensegric structure

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Abstract: The present paper promotes a shape optimisation method for maximum overall stiffness of a tensegric structure, and reports a study on the overall stiffness of a tensile structure. General reduced gradient method is adopted for the shape optimisation, and a tensegric dome is taken as an example for the numerical demonstration. The results of numerical analysis show that the nodal displacements of the example decreases from 20%–30% compared to that of the initial shape.

Keywords: shape optimisation; tensegrity; tensile structure; cable dome; minimum displacement; maximum stiffness; overall stiffness.

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Biographical notes: Pei-Shan Chen received his DrEng at the House University. He is a Professor at the Department of Civil Engineering and Architecture as well as Graduate School in the Hachinohe Institute of Technology (HIT). Before teaching in the HIT, he had worked as an Engineer in Chief for more than ten years and designed many big span spatial structures and high-rise buildings. Now, his research interests are related to shape optimisation, creating new structure system, seismic design, and historical structures. He is an author of a great deal of research studies published at national and international journals, conference proceedings.

1 Introduction

A tensile structure is known as a flexible structure, especially when it is deformed against asymmetrical loads as well as wind loads. To achieve high stiffness in a tensile structure, therefore, high prestressed forces in members are necessary, which usually negatively affects the cost and method of construction. However, the structural performances of a tensile structure are heavily dependent on its shape and prestresses introduced in the structure. Such a structural feature makes it very important to maximise the overall stiffness of a tensile structure by optimising its shape and distribution of prestressed forces through a mathematical programming, and that has been the research subjects of several former studies of the author (Chen et al., 1993; Chen and Kawaguchi, 1993).

squares of all vector elements. Considering the equilibrium equation [equation (6)], the author promotes a function below as the stiffness evaluation function.

$$\begin{aligned} F &= \mathbf{D}^T\mathbf{D} = \mathbf{Q}^T\left(\mathbf{K}^{-1}\right)^T\mathbf{K}^{-1}\mathbf{Q} \\ &= \mathbf{Q}^T\overline{\mathbf{K}}\mathbf{Q} \end{aligned} \tag{8}$$

where $\overline{\mathbf{K}} = \left(\mathbf{K}^{-1}\right)^T\mathbf{K}^{-1}$. Equation (8) is a sum of positive values $D_i^2 \geq 0$, which is very different from that presented in the last section.

In order to discuss if such an evaluation function results in a similar conclusion as equation (7), let us again assume that the first element of the load vector is zero, $Q_1 = 0$. Consequently, the equation (8) can be rewritten as the equation below.

$$\begin{aligned} F &= \left(0, Q_2, \ldots, Q_n\right)\begin{bmatrix} \overline{K}_{11} & \cdots & \overline{K}_{1n} \\ \vdots & \ddots & \vdots \\ \overline{K}_{n1} & \cdots & \overline{K}_{nn} \end{bmatrix}\begin{pmatrix} 0 \\ Q_2 \\ \vdots \\ Q_n \end{pmatrix} \\ &= \sum_{i=2}^{n}\sum_{j=2}^{n} Q_i\overline{K}_{ij}Q_j \end{aligned} \tag{9}$$

Then, we get the similar conclusion as equation (7). However, the present methods can also be used to evaluate the vertical stiffness of a tensegric structure. In common, the shape of a tensegric structure is determined by radii and rises, and such geometric dimensions affect the nodal displacements including the components where the load components are zeroes. So, the present method is efficient for the shape optimisation for maximum overall stiffness.

If equation (8) is regarded as a virtue work and weighting factor method presented in Subsection 2.1, by using equation (1) and assuming, $\psi_i = D_i$, an important result can be found that a big displacement component acts as a heavy weighting factor while a small component acts as a light one, such that evaluation function (8) provides heavy weighting factors acting on the big displacements, $F = \sum\psi_i D_i = \sum D_i D_i$. Then, we can again infer that the present method is efficient for the evaluation of overall structural stiffness.

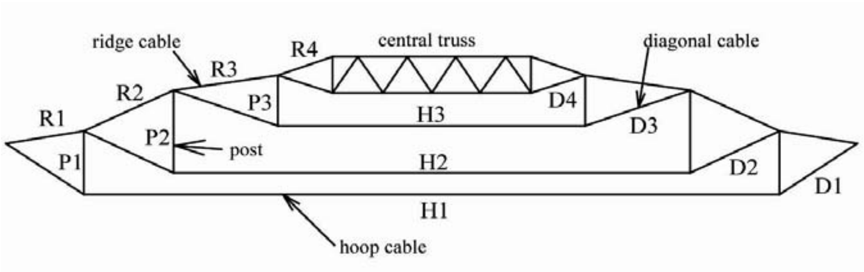
3 Optimisation theory for a tensile structure

3.1 The mathematical analysis model

In this study, the author promotes an optimisation method for maximum overall stiffness of a tensegric structure. Based on the studies in former sections, the author chooses the minimum displacement method, demonstrated in Subsection 2.3, to evaluate the overall stiffness of the structure.

For a tensegric structure with n degrees of freedom, the objective function F_1 for minimum displacements can be expressed as the following equations.

Figure 3 Member symbols



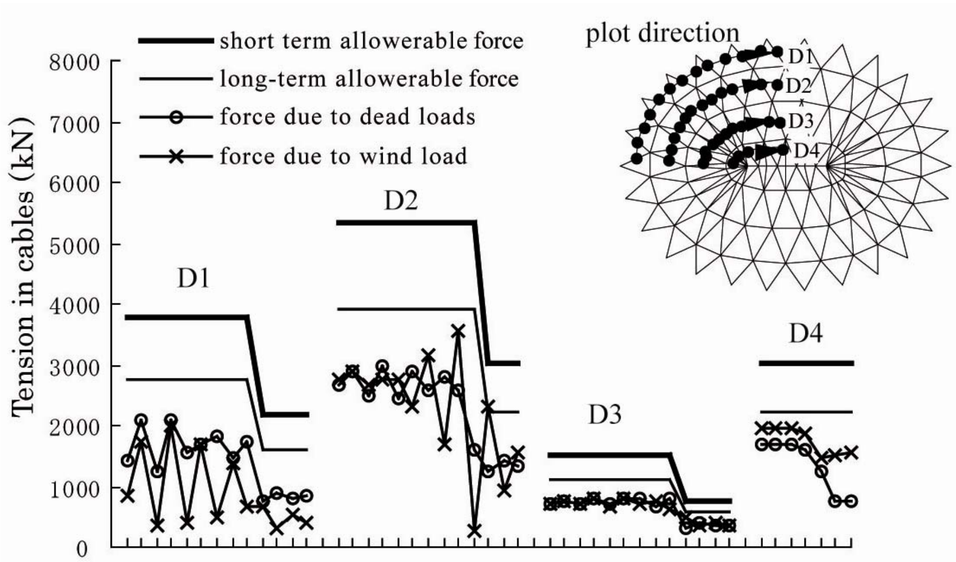
A distributed load of 300 N/m^2 was assumed to be the dead load applying downwards at the top of the roof. For the wind loads, a design velocity load of 2.52 kN/m^2 and a wind force coefficient (for entire roof surface, upward) of C = 0.8 were assumed.

Table 1 Materials of major members

Symbols	*Material*	*Symbols*	*Material*
R1	2 – ϕ88 + 2 – ϕ53 + 2 – ϕ50	D1	2 – ϕ67 + 1 – ϕ60
R2	2 – ϕ53 + 4 – ϕ50	D2	2 – ϕ88
R3	2 – ϕ53 + 2 – ϕ50	D3	2 – ϕ50
R4	4 – ϕ50	D4	4 – ϕ50
H1	5 – ϕ100	P1	ϕ – 609.6 × 22.0
H2	2 – ϕ88 + 6 – ϕ100	P2	ϕ – 711.2 × 22.0
H3	4 – ϕ88	P3	ϕ – 508.0 × 22.0

Note: Member symbols are referred in Figure 3.

Figure 4 Prestresses in diagonal cables



1 to evaluate the virtual work done by a virtual load set

2 to evaluate the magnitude of the displacement vector.

Both of the promoted methods are efficient for the optimisation for maximum overall stiffness of a tensegric structure.

The present paper demonstrated an optimisation method for maximum overall stiffness of a tensegric structure. By the numerical analysis, the author demonstrated that the present optimisation is an effective method to find the optimum shape with maximum stiffness. Consequently, the numerical analysis results show that the vertical displacements decreased 20–30% comparing to that of its initial shape. The present optimisation can determine the member sections automatically and output more useful information for design decisions.

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